

Small-world and Scale-free Properties of Urban Street Networks: A Tutorial

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Introduction

This tutorial provides a step by step guide for calculating some measures of urban street networks in order to examine if they possess small-world and scale-free properties (Jiang and Claramunt, 2004). The urban street network of the French city Avignon is used as an example. A graph is characterized as a small-world if it is highly clustered locally and with a small separation globally, which are measured respectively by clustering coefficient and average path length (Watts and Strogatz, 1998). In addition, different measures of centrality (degree, closeness and betweenness) are used to determine the relative importance or status of a node within the graph. Barabási and Albert (1999) pointed out the emergence of scaling in random networks in which the degree distribution of large networks follows a power-law distribution, i.e., so called scale-free networks. In other words, a scale-free network is the network whose degree distribution fits a power-law distribution. In this tutorial, we adopt some very restrict means to detect the power-law behavior in empirical data, developed by Clauset, Shalizi and Newman (2009). This tutorial to a great extent aims to replicate similar investigation conducted by Jiang and Claramunt (2004).

As mentioned above, small-world property is mainly characterized by two measures: average path length on the one hand and clustering coefficient on the other hand. Average path length is the average number of steps between the nodes of a graph and these steps are along the shortest paths. Clustering coefficient is used to describe the clustering degree, i.e., the friends of a friend are likely to be friends. These measures could be applied to make topological analysis of urban street networks, after they are transformed into a graph, i.e., lines as node, and line intersections as links of the graph. Three measures of centrality include degree, closeness and betweenness. Degree is connectivity in other term and it describes the number of links that a node has. Closeness centrality can be regarded as the measure of distance from one node to all other nodes. Betweenness records the number of times a node is used as a bridge for other two nodes along the shortest paths. These centrality measures can determine how important a node in the graph and examine if the urban street network has scale-free property.

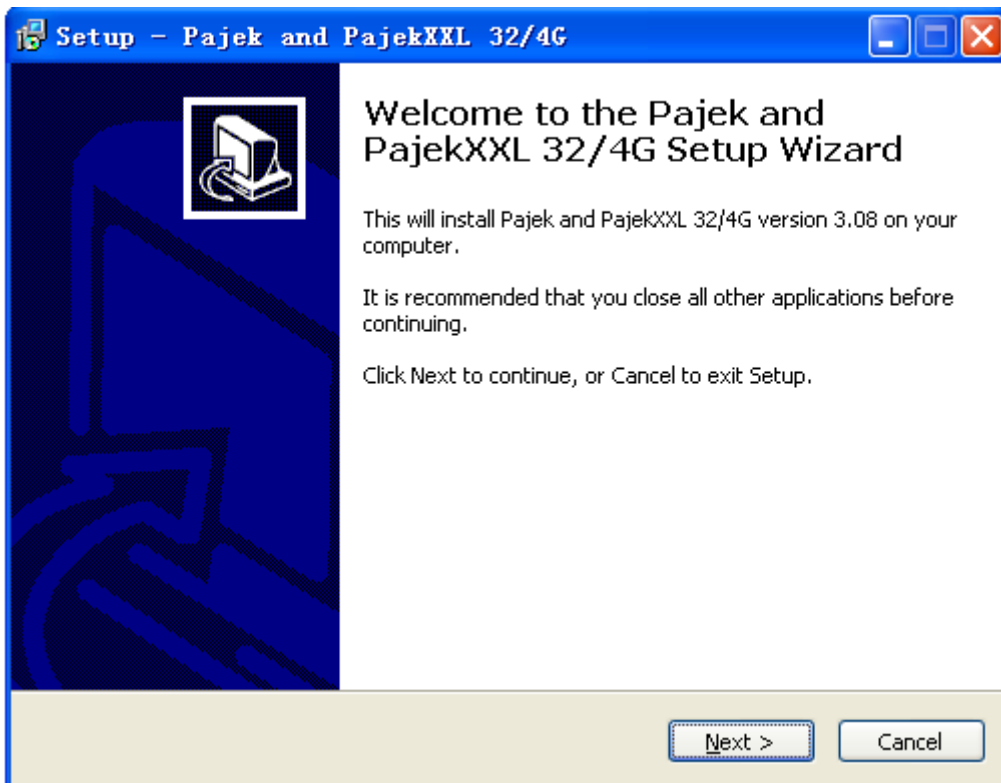
All these measures are computed relying on Pajek, a free software for large network analysis and visualization. In this tutorial, we assume that you have used axial map to represent urban street networks rather than named streets. For how to automatically generate axial map or axial lines, one can refer to Sun (2012) for details. It is also assumed that you have successfully converted axial lines shape file to Pajek's net file. MATLAB software is used to check if the degree distribution of the urban street network fits power-law distribution.

The reminder of the tutorial is structured as four steps: installing Pajek and MATLAB software and downloading Clauset's MATLAB code, importing data and calculating computational measures, examining small-world properties and discerning scale-free properties of the urban street network of Avignon. Make sure you understand these measures for analyzing urban street networks.

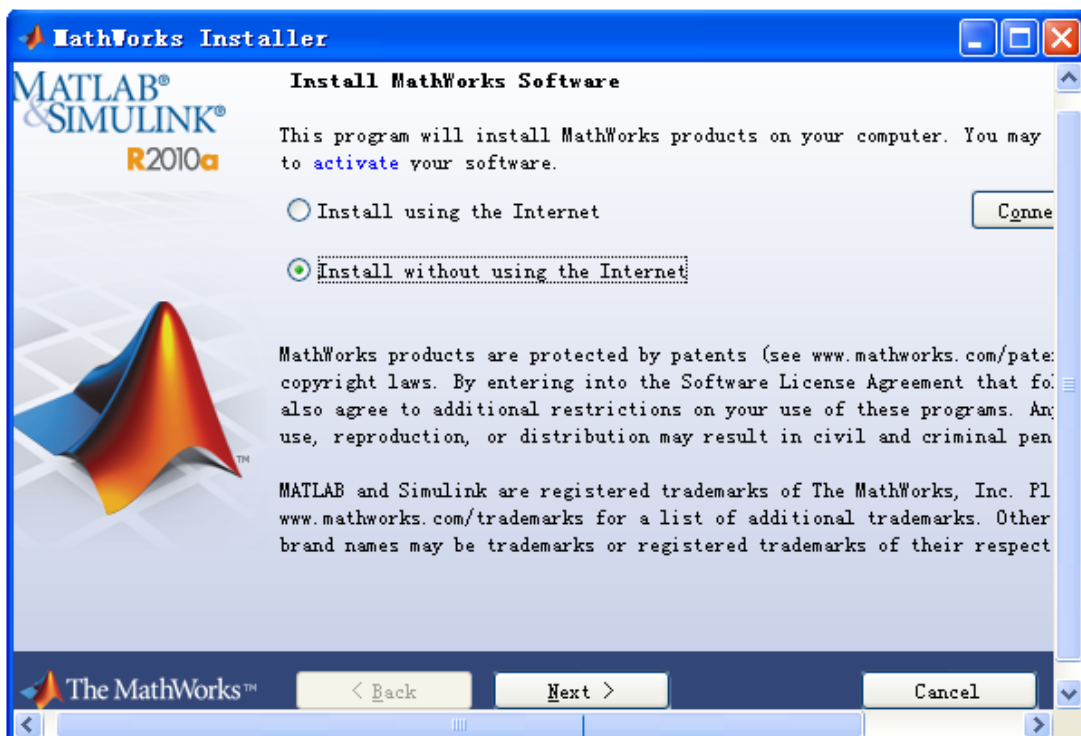
Install Pajek and MATLAB and download Clauset's MATLAB code

1. Go to the website <http://pajek.imfm.si/doku.php?id=download> to choose an appropriate Pajek version for your computer. Click the link to download the zip file.
2. After downloading, unzip the exe file and double click it to install Pajek. Click *Next* when a setup window pops up. Click *Next* and choose a folder to store the software and click *Next* again. Finally click

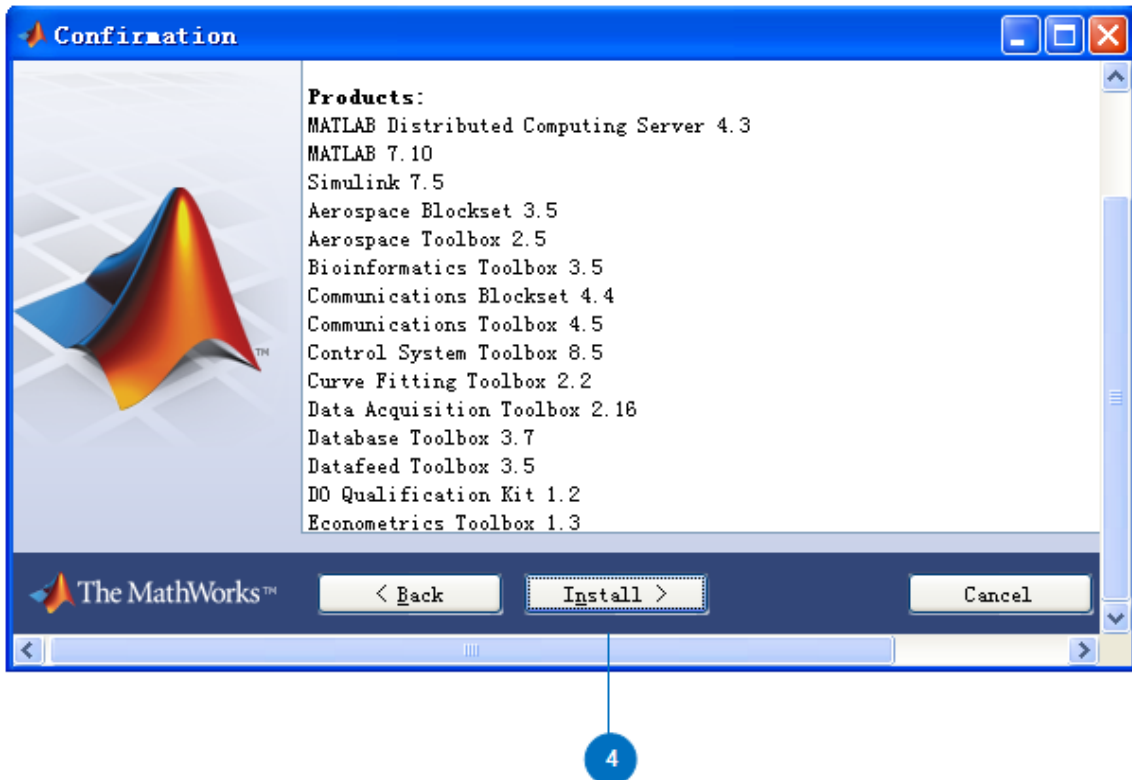
Install to continue the installation.



3. Search “download MATLAB” through websites and download the software. Find the setup.exe file in the downloaded folder and double click it to start installation. Click *Next*, choose a folder to store it and keep clicking *Next* until get the confirmation window.



4. Click *Install* after confirming the installation settings.



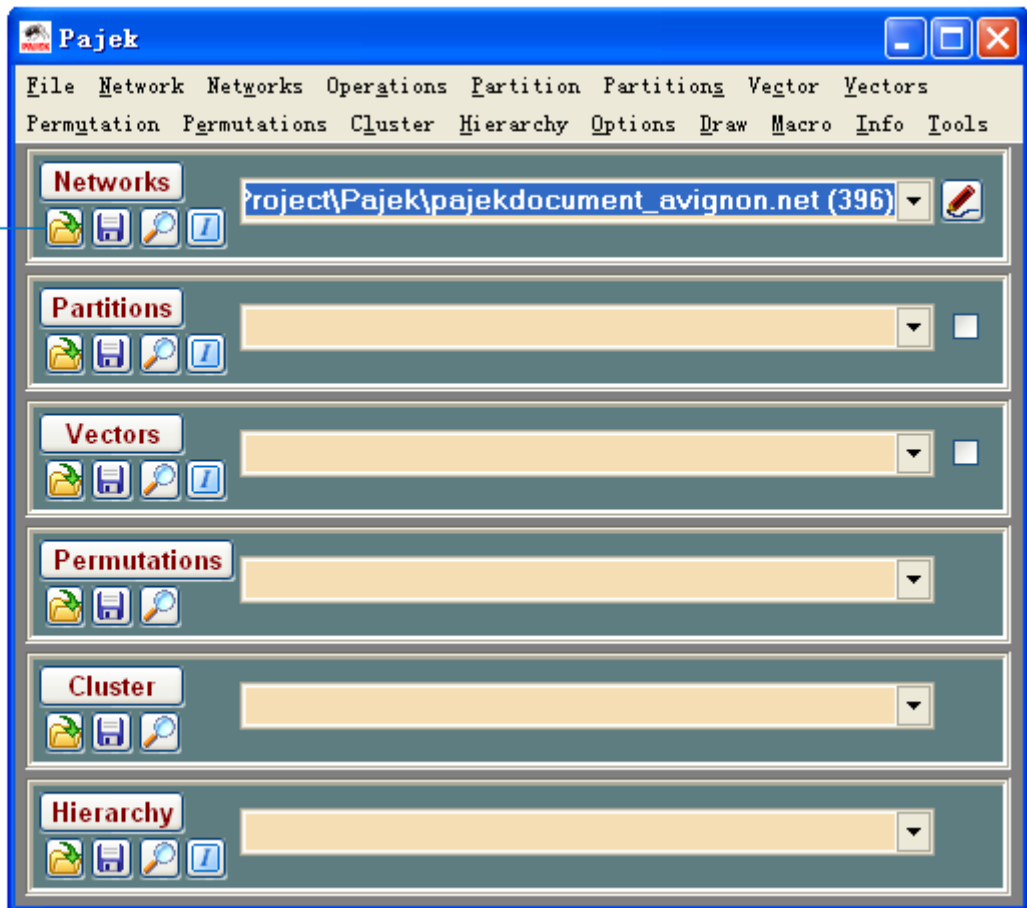
5. To discern power-law distribution in MATLAB, Clauset MATLAB code should be downloaded. Go to <http://tuvalu.santafe.edu/~aaronc/powerlaws/>, find `plfit.m`, `plplot.m` and `plpva.m` and save them as MATLAB code files into your created matlab folder.

Import data and calculate computational measures

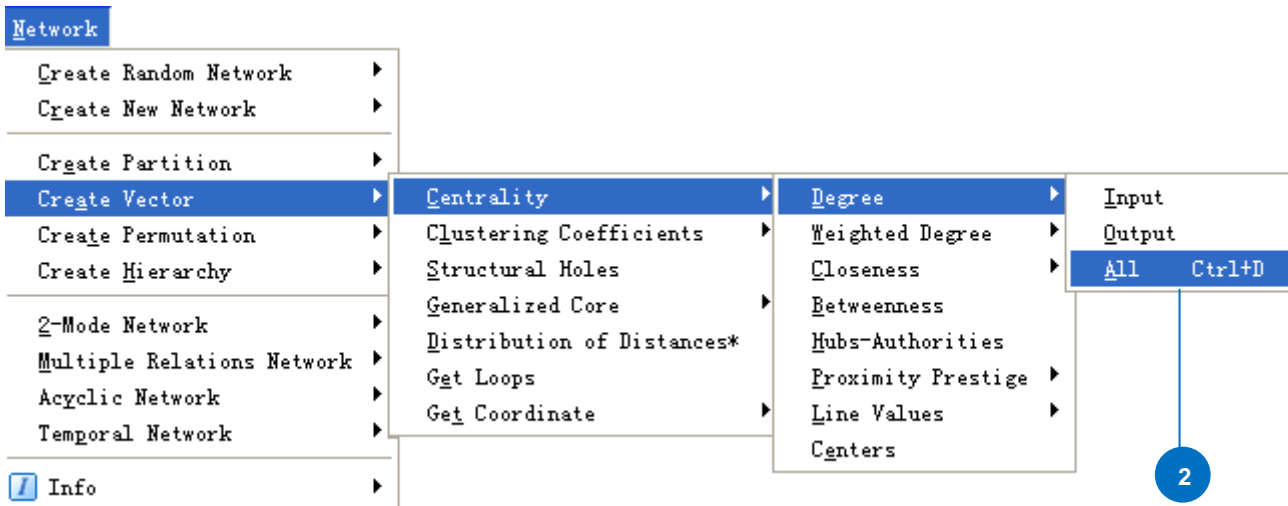
To judge if an urban street network has scale-free properties, the value of degree, closeness and betweenness should be considered. Pajek provides a way to compute these values. To examine if an urban street network is a small-world network, average path length and clustering coefficient need to be calculated. These two values can be measured through Pajek.

Calculate centrality (degree, closeness and betweenness)

1. Start Pajek and click *Read Network* button under networks field to open the converted Avignon net file.



2. Click *Network* in the menu and then choose *Create Vector, Centrality, Degree* and click *All* to get the degree values of urban street network of Avignon. The “degree” is same as the term “connectivity” gotten in ArcGIS.

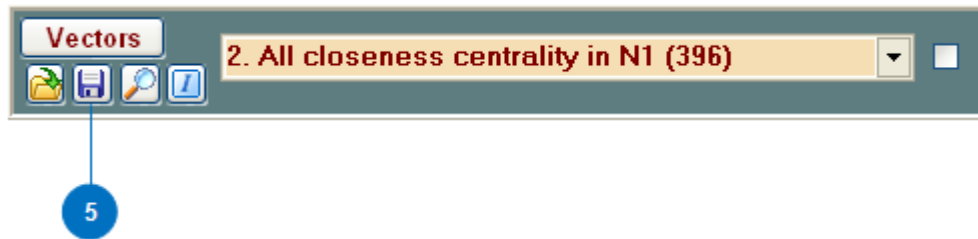


3. Click *Save Vector* button in the vector field to save the computed degree vector of the network.

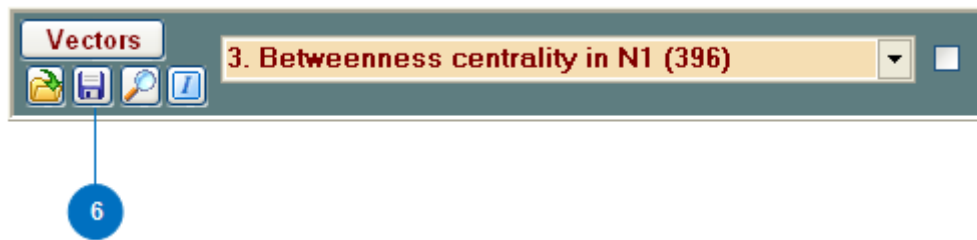


4. Find the folder where you save the degree vector file and use Notepad to open this file.
5. Calculate closeness of the urban street network with steps that are similar to the procedures of calculating degree. Click *Network* in the menu, choose *Create Vector, Centrality, and Closeness* and click

All. Click *Save Vector* button to save the closeness value.

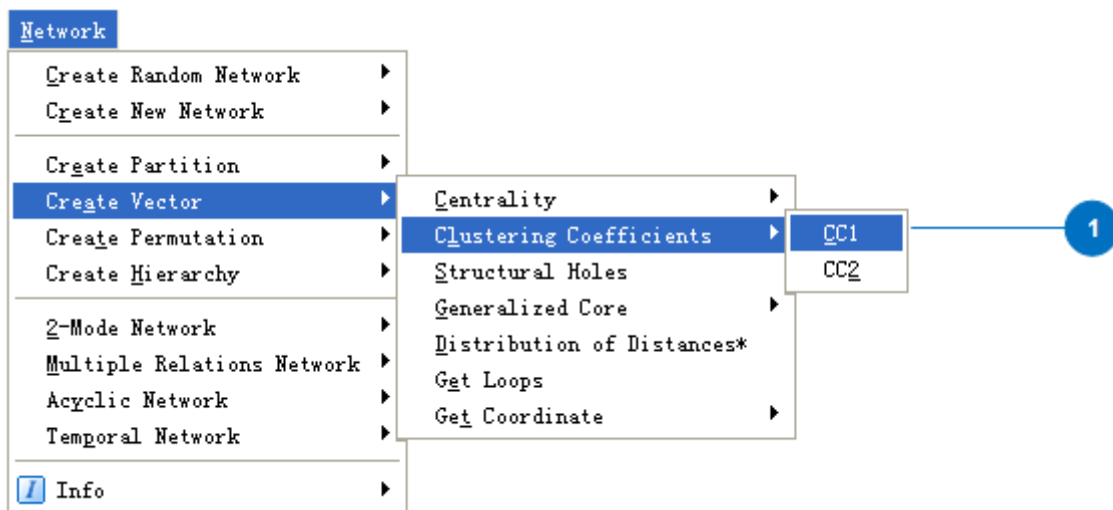


6. Measure betweenness of the network as previous steps. Click *Network* in the menu, choose *Create Vector*, *Centrality*, and click *Betweenness*. Click *Save Vector* button to save the betweenness value.

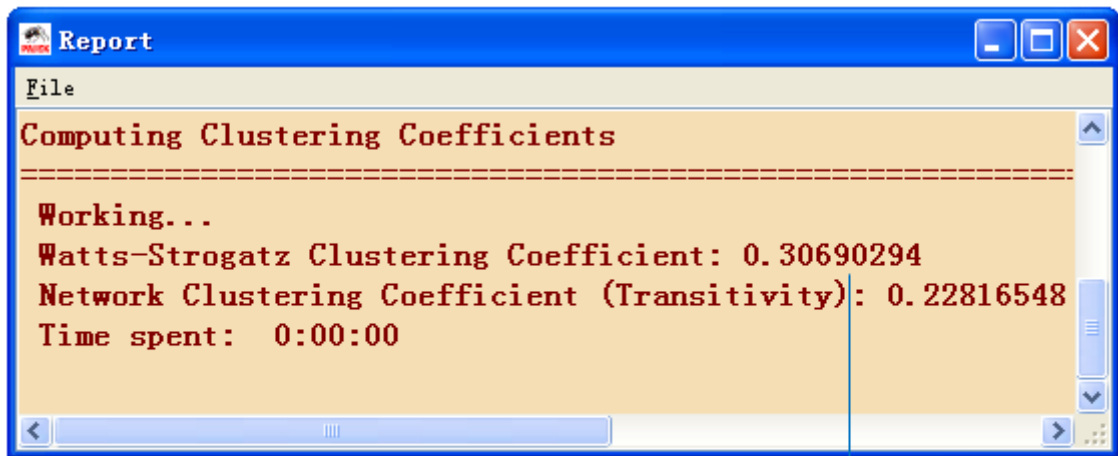


Calculate average path length and clustering coefficient

1. Click *Network* in the menu, choose *Create Vector* and *Clustering Coefficients* and click *CC1* to get clustering coefficient of the urban street network. You can also calculate CC2 following the same procedures.



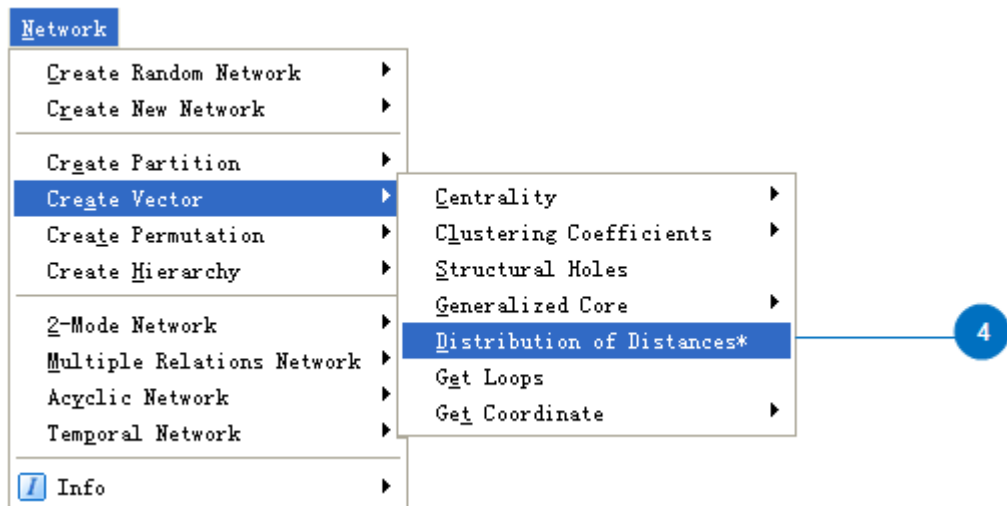
2. Notice the popped up report window and record the computed clustering coefficient in the report.



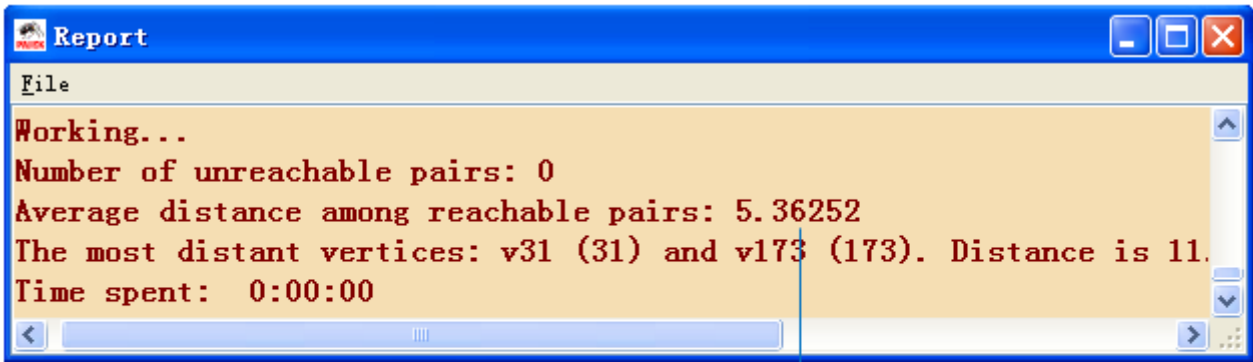
3. Click *Save* button in each field to store the generated files about clustering coefficients.



4. Click *Network* in the menu, choose *Create Vector* and click *Distribution of Distances* to get average path length of the network.



5. Notice the popped up report window and record average distance among reachable pairs.



6. Click *Save* button to save the vector file of distribution of distances.



Examine small-world properties of urban street network of Avignon

Small-world network is the network that most members which are not neighbors can be reached from every other with a small number of steps. Six degrees of separation is an idea applied for small-world network. To examine if the network has small-world properties, average path length (L) and clustering coefficient ($CC1$) value are considered.

1. Open the file of degree and compute the mean value (\bar{m}).
2. For comparison purpose, the measures of average path length (L_{random}) and clustering coefficient (C_{random}^1) for a random graph with the same degree per vertex and same number of nodes are calculated. $L_{\text{random}} = \ln n / \ln \bar{m}$, $C_{\text{random}}^1 = \bar{m} / n$, where n is the total number of vertices.

Calculation results for Avignon city:

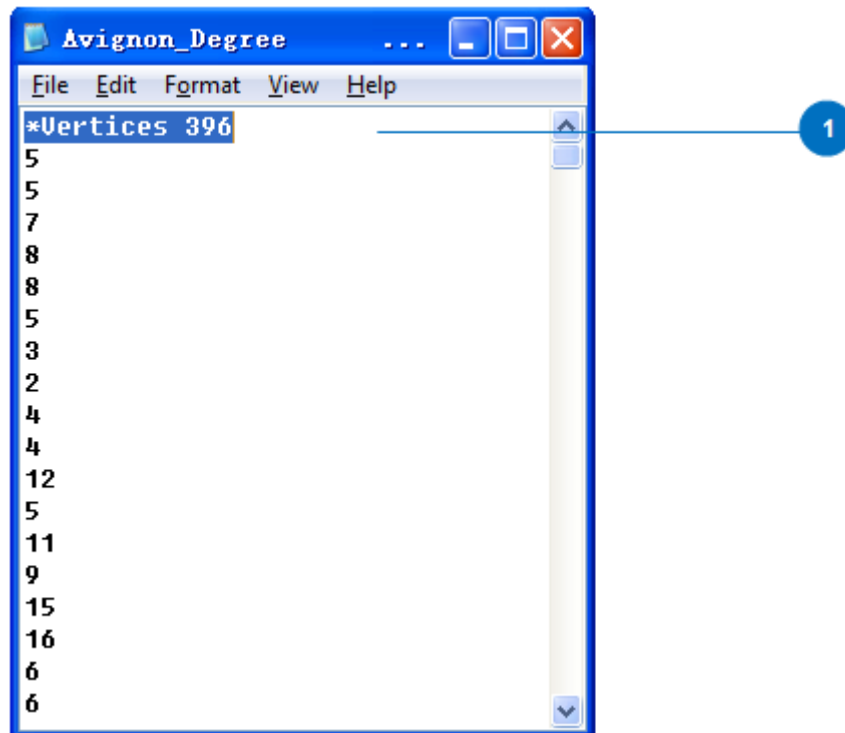
\bar{m}	L_{actual}	L_{random}	C_{actual}^1	C_{random}^1
5.5303	5.3625	3.4974	0.3069	0.0139

The calculated results show that the street network of Avignon has small degrees of separation and the clustering coefficient meets the condition that $C_{\text{actual}}^1 \gg C_{\text{random}}^1$. These measures reflect that the street network is a small-world network.

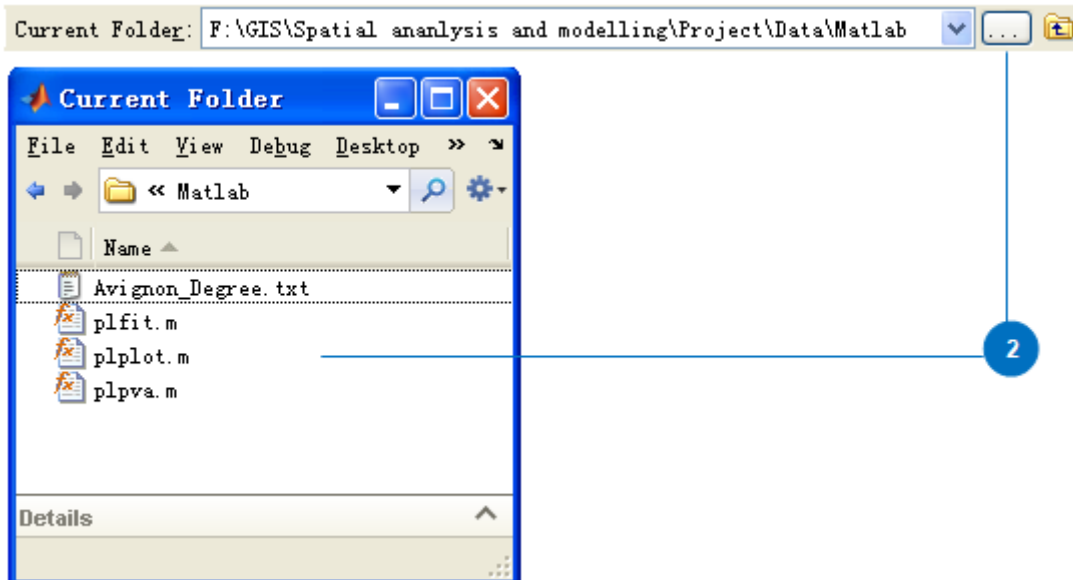
Discern scale-free properties of the urban street network

Scale-free network has the characteristic that the degree distribution is a power-law distribution. To find if the street network is scale-free network, the measure of degree value is used.

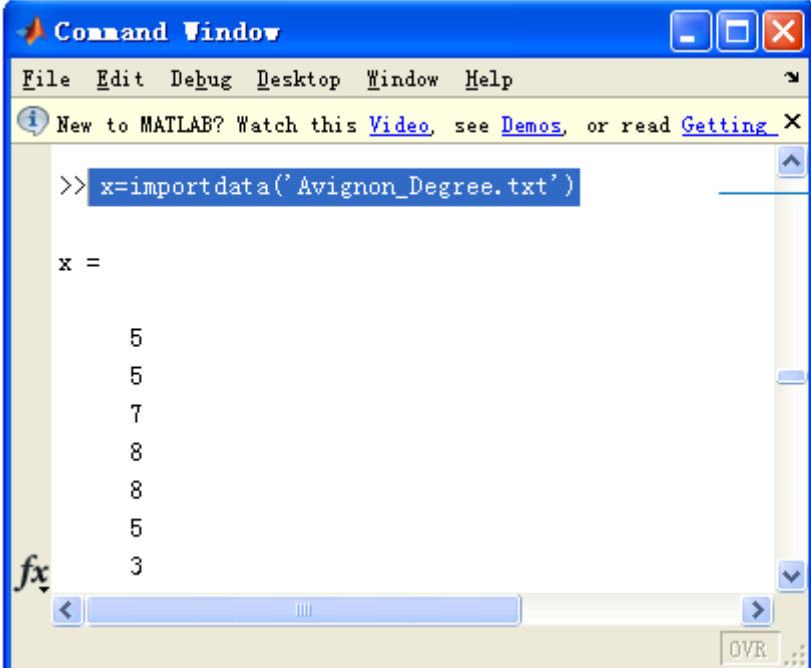
1. Use Notepad to open these vector files about the measures of centrality. Delete the first line in each file and save them as text format (*.txt).



2. Open MATLAB and click *Browse for folder* button to find the folder you are working with. **Note:** Make sure this folder contains your saved degree, closeness and betweenness text files and Clauset MATLAB code plfit.m, plplot.m and plpva.m files. These three MATLAB code files have the functions to check if the data fit a power-law distribution.



3. In the command window, type in the import data code and press *Enter* on the keyboard.



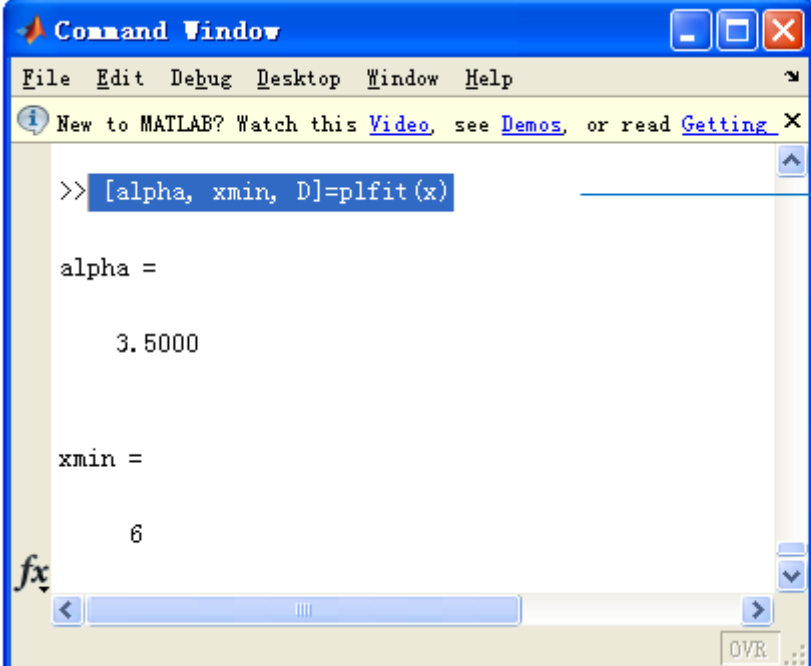
The image shows a MATLAB Command Window with a blue title bar and a menu bar (File, Edit, Debug, Desktop, Window, Help). A message at the top says "New to MATLAB? Watch this Video, see Demos, or read Getting...". The command prompt shows the execution of `x=importdata('Avignon_Degree.txt')`. The output displays the variable `x` as a column vector of values: 5, 5, 7, 8, 8, 5, 3. A blue circle with the number 3 is positioned to the right of the command line.

```
>> x=importdata('Avignon_Degree.txt')

x =

     5
     5
     7
     8
     8
     5
     3
```

4. Type power-law fit function in the command window and get the alpha and minimum value. Press *Enter* after finishing editing the code.



The image shows a MATLAB Command Window with a blue title bar and a menu bar (File, Edit, Debug, Desktop, Window, Help). A message at the top says "New to MATLAB? Watch this Video, see Demos, or read Getting...". The command prompt shows the execution of `[alpha, xmin, D]=plfit(x)`. The output displays the variable `alpha` as 3.5000 and the variable `xmin` as 6. A blue circle with the number 4 is positioned to the right of the command line.

```
>> [alpha, xmin, D]=plfit(x)

alpha =

     3.5000

xmin =

     6
```

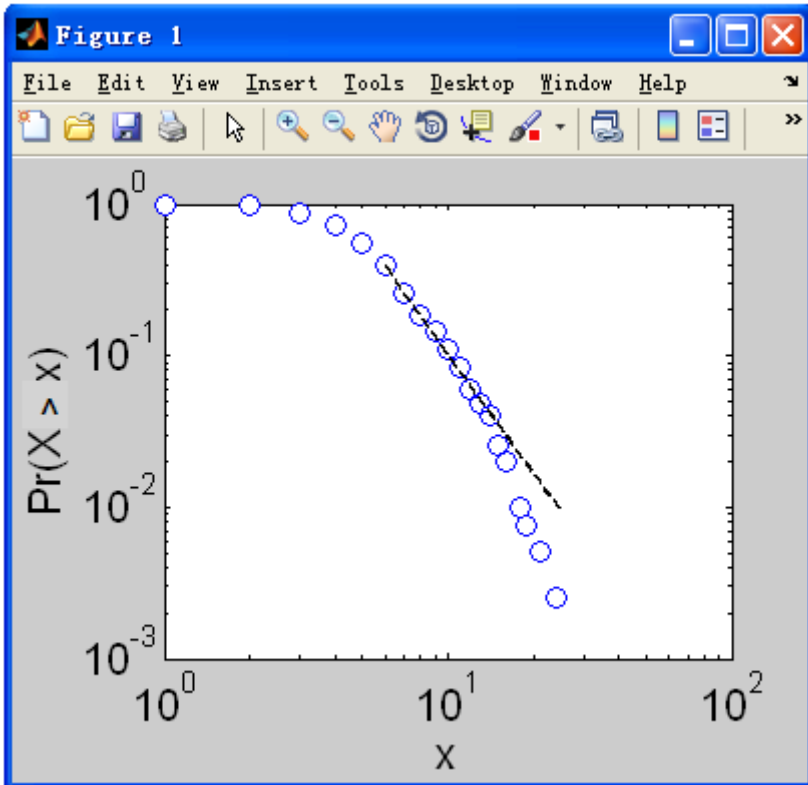
5. Next, use `PLPLOT` function, type in the code and press *Enter*. You will get a figure that shows the power-law distributional model with Avignon's data. You can check the plot and find how well it fits a power-law distribution.

```

Command Window
File Edit Debug Desktop Window Help
New to MATLAB? Watch this Video, see Demos, or read Getting
174.0016
175.0011
>> h=plplot(x, xmin, alpha)
h =
352.0016
353.0011
fx >>

```

5



- Use PLPVA function to calculate the p-value for the given power-law fit to Avignon's data and type the code in command window. You can choose different calculating methods and they can be found in plpva.m file by double clicking the file in MATLAB. Press *Enter* after typing the code and you will get the p-value at the end.

```

Command Window
File Edit Debug Desktop Window Help
New to MATLAB? Watch this Video, see Demos, or read Getting
>> [p,gof]=plpva(x, xmin, 'reps', 500)
Power-law Distribution, p-value calculation
Copyright 2007 Aaron Clauset
Warning: This can be a slow calculation; please
n = 396
xmin = 6.0000
reps = 500
[1] p = 0.0000 [0.01m]
[2] p = 0.5000 [0.01m]
[3] p = 0.6667 [0.01m]
[4] p = 0.5000 [0.01m]

```

6

```

Command Window
File Edit Debug Desktop Window Help
New to MATLAB? Watch this Video, see Demos, or read Getting
[498] p = 0.4819 [0.15m]
[499] p = 0.4830 [0.15m]
[500] p = 0.4820 [0.15m]
p =
0.4820
gof =
0.0392

```

If a p-value which is larger than 0 is gotten, the tested data could fit a power-law distribution. For Avignon, the p-value is equal to 0.4820 that means the street network has scale-free properties. You can also check the closeness distribution and betweenness distribution of Avignon with same procedures to see if they fit power-law distribution.

After go through this tutorial, you have learned how to examine an urban street network and find out if it is a small-world network and a scale-free network. Degree, closeness, betweenness, average path length and clustering coefficient of a street network can be calculated from Pajek. These steps are applied to the converted net file of Avignon's street network and you can follow this tutorial to examine the street network of your own data.

Acknowledgement:

This tutorial is developed as part of a course project. I would like to thank Prof. Bin Jiang for his guidance.

References

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