

# Visualisation of Fuzzy Boundaries of Geographic Objects

Bin Jiang

*The linguistic notions such as very low, low, not low, low or medium are commonly used to name classes in current geographic information systems (GIS). These sorts of linguistic notions are frequently utilised in the social sciences as well. There is little doubt that the social sciences require a formal and even a mathematical framework for handling graded categories with blurred boundaries. In the past decades, much effort has been made to model the kind of fuzziness (or possibility) from the field of mathematics. Geographers and GIS professionals have started to treat this issue since a decade ago. This short article provides detailed discussions with a case study on how to visualise fuzziness, involving colour surfaces, coloured contour lines and 3D model simulation. The author's argument is that effective visualisation of fuzzy boundaries will facilitate the understanding of geographic objects with indeterminate boundaries.*

*An earlier version of this paper was presented at the Workshop on Graphics, Visualisation and Social Sciences, 8 & 9 May 1997, Burleigh Court, Loughborough University.*

**Bin Jiang**  
Centre for Advanced Spatial Analysis  
University College London  
1-19 Torrington Place, WC1E 6BT, London  
Email: b.jiang@ucl.ac.uk

## INTRODUCTION

The linguistic notions such as *very low, low, not low, low or medium* are commonly used to name classes in current geographic information systems (GIS). These sorts of linguistic notions are frequently utilised in the social sciences as well. There is little doubt that the social sciences require formal and even mathematical frameworks for handling graded categories with blurred boundaries (Smithson, 1988). In the past decades, much effort has been made to model this kind of fuzziness (or possibility) from the field of mathematics. Geographers and GIS professionals have been investigating this issue for a decade (Burrough, 1989).

What is basic to the definition of fuzzy boundaries of geographic objects is the membership function, which serves as one of the fundamental parts of Fuzzy Logic (Zadeh, 1965). The membership function of a fuzzy set defines how the grade of membership of  $x$  in  $A$  is determined, for crisp sets the membership function has only two values: 0 or 1. Different from that of crisp sets, the fuzzy membership function aims to measure something between 0 and 1. However, this article does not cover the measuring issue, which has been treated elsewhere (e.g. Burrough and Frank, 1996; Jiang and Kainz, 1996).

Instead of measuring fuzzy boundaries, this article intends to provide a discussion on how to visualise fuzzy boundaries by the use of colour

variables, as well as a 3D simulation model. It is the author's belief that effective visualisation schemes will benefit the understanding and perception of geographic objects.

Over the past years, colour variables have been widely used in the visualisation of uncertainty or data quality (e.g. Schweizer and Goodchild, 1992; Brown and van Elzakker, 1993; Jiang *et al.*, 1996). More general issues are addressed such as the impediments to representing data quality (Buttenfield 1993), cartographic views to spatial data quality (e.g. McGranaghan, 1993; Beard and Mackaness, 1993). In addition, many multimedia variables including sounds and animation have been discussed for the visualisation of uncertainty (Fisher, 1994; 1996).

This article focuses on two principles of using colour in the visualisation of fuzzy boundaries. In section two, colour variables and colour scales are introduced which could be used for visualisation of fuzzy boundaries. Then in section three, principles of visualising fuzziness are presented. After that, a case study on the definition of town centres is reported, and some examples are shown before drawing conclusions.

## COLOUR VARIABLES AND COLOUR SCALES

Colour is one of the important visual variables as represented, for example, in Bertin's (1983) visual system. It has long been used for representing qualitative and quantitative data (Tufte, 1990; Dent 1996). Following is a brief introduction to three fundamental colour variables for the purpose of visualising fuzzy boundaries of geographic objects.

*Hue* is what is meant when referring to *colours* by names such as red, green, blue etc . . . In Bertin's visual variable system, hue is referred to as colour. From the point of view of physics, it is the visual stimulus provided by a dominant single wavelength, or by a dominant mixture of wavelengths.

This colour variable has an associative perception property. Therefore, it can be used very effectively in the representation of qualitative information, for instance, the difference between soil classes in a soil map.

There are some generally accepted associations or conventional assignments of colours to specific attributes, e.g. green to vegetation, red to high temperature.

*Saturation* is another colour sensation, which is not included in Bertin's system of visual variables because it is hard to perceive and very hard to vary in practice, particularly in a paper map. Saturation is given to describe the relative purity of colour, by adding different percentages of grey.

*Lightness* is included in Bertin's system, where it is referred to as value. This colour variable refers to the relative lightness or darkness of a symbol, and has ordered and selective perception properties, so it is often used to represent ordered or relative quantitative information.

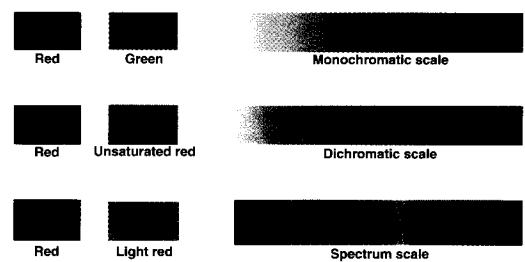


Figure 1. Examples of colour variable and colour scales

With the above colour variables, it is possible to generate different colour scales to represent qualitative and quantitative information. A colour scale is a range of colours with variations of colour variables. A range of colour scales could be generated for the representation of different sorts of information. For instance, the red scale shown in Figure 1 can be used to colour average annual temperature. In the following section, a customised colour scale is introduced, which the author feels is more intuitive from the point of view of colour perception.

## PRINCIPLES OF VISUALISING FUZZY BOUNDARIES

Colour is a useful tool to be used for the visualisation of fuzzy boundaries. It was also observed by Zadeh (1973) that *If we regard*

the colour of an object as a variable, then its values, red, blue, yellow, green etc. may be interpreted as labels of fuzzy subsets of a universe of object. Within the context of visualising fuzzy boundaries of geographic objects, each object may be identified as a certain colour hue, with purest colours in the centre surrounded by non-purest colours. This kind of visualisation gives the users the impression of fuzzy objects, with colour purity or density representing different uncertainties (Figure 2). Therefore, the principle is *the purer, the more certain, or the denser, the more certain*.

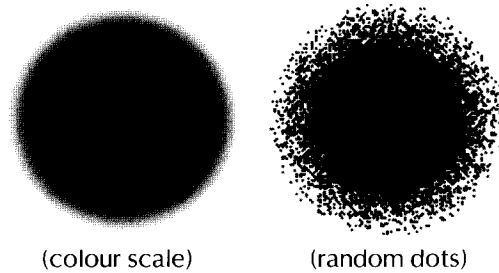


Figure 2. Visualisation of fuzzy boundaries

Another principle in visualising fuzzy boundaries is a so called *visual equality*. In other words, colours representing values equally different from each other along the scale should be perceived equally different. This can be described as follows,

for any  $1 \leq i, j, m, n \leq N$ ,

if  $v_i - v_j = v_m - v_n$  then

$pd(c_i, c_j) = pd(c_m, c_n)$

where  $i, j, m, n$  are four sequential colour indexes in a colour scale with  $N$  number of colours,  $v$  is the value that the colour represents,  $c$  is the colour value of itself, and  $pd$  is the perceived distance between two adjacent colours. In the next section, a colour scheme with visual equality will be presented.

### CASE STUDY

The case study is from the Definition of Town Centres project funded by the UK Department of Environment (DoE). The aim of the project is to develop geographical definitions of town centres for both statistical and planning purposes. What is the town centre? According

to PPG6 (DoE 1996), the town centre is defined as including *the city, town and suburban district centres, which provide a broad range of facilities and services and act as focus for both the community and for public transport. It excludes small parades of shops of purely local significance*. Literally *centre* is quite a fuzzy concept, which can only be defined as a fuzzy surface.

It is recognised that it is hard to draw a boundary to delineate the town centre and non-town centre, and actually no clearcut boundary exists. Therefore, from the point of view of fuzzy logic, the town centre can only be represented as a fuzzy surface. The basic approach to defining the town centre is fuzzy overlay analysis (Jiang and Kainz, 1996). There is a range of factors to be taken into account for the definition of town centres, for instance, population, activities and facilities, diversity, the demand for space, and measures of accessibility. For each factor, an individual town centre could be defined. Overlaying all of these individual town centres leads to a composite town centre.

In Figure 3, some visualisation schemes are shown by using data from Wolverhampton. Wolverhampton was chosen as the case study because there is a ring road surrounding the central town, which can be roughly thought of as the boundary of the town centre. The individual fuzzy surfaces are visualised by the use of monochromatic and dichromatic colour scales. The linear scale means that the fuzziness( $x$ ) of what each colour represents has a linear relationship with the colour value( $y$ ) itself, e.g.  $y = kx + b$ , while the curve means a curved relationship between fuzziness and colour value, e.g.  $y = kx^2 + b$ . It is easy to interpret that, in monochromatic scale, purest red is the town centre with 100% certainty, white is the town centre with 0% certainty (or the non-town centre), and other reds represent something between town centre and non-town centre. The same interpretation is made in the dichromatic colour scale except where non-town centre is represented by yellow instead of white. The immediate impression is that curved colour scales might hold visual equality, but not for the linear colour scales. This claim should be tested before drawing a conclusion.

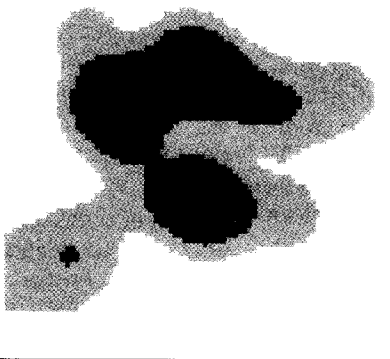
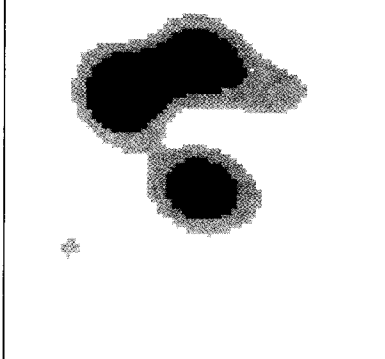
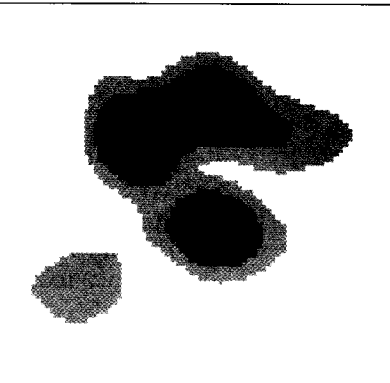
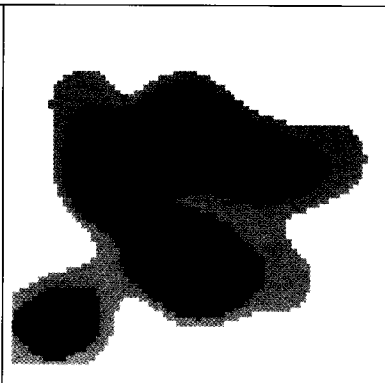
	Linear	Curved
Monochromatic		
Dichromatic		

Figure 3. Illustration of linear and curved scales

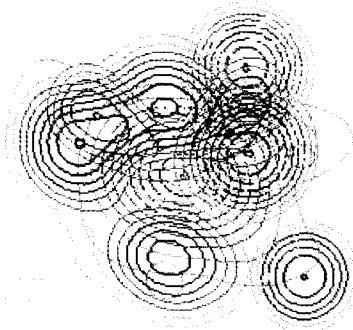


Figure 4. Visualisation of individual fuzzy surfaces using contours

Not only can colour scales be used for visualisation of fuzzy surfaces, but contours and 3D models can as well. Figure 4 is an example where each fuzzy surface is represented as a group of contour lines indicated by the unique colour (hue). With the

visualisation as shown in Figure 4, it is easy to explore the topological relationships of fuzzy surfaces. Figure 5 is an example of a 3D model which is generated using VRML. It is possible to walk and fly through the model or change the perspective of viewing. It displays the certainty value while the mouse pointer passes through.

## CONCLUSIONS

This article has discussed issues of visualising fuzzy boundaries of geographic objects by the use of colours. It has discussed the basic principles and demonstrated these using a case study. It shows that colour is one of the intuitive ways of visualising fuzzy boundaries. It also illustrates that the curved colour scales might hold visual equality, although it needs perception testing before any conclusive answer can be made. The simulation model



Figure 5. Perspective view of 3D model for the fuzzy surface

sets a good example for visualisation using animated cartography (Ormeling, 1995). Fuzziness of geographic objects is getting increasing attention in the process of spatial decision making. Visualisation of fuzziness will benefit visual decision making (Kraak *et al.*, 1995) as an alternative to traditional cartographic displays.

#### ACKNOWLEDGEMENT

The author would like to thank Sarah Sheppard for revising on version of this paper, and Martin Dodge for providing the model shown in Figure 5. Two anonymous reviewers and Graeme Wright provided useful comments. Thank you.

#### REFERENCES

- Beard K. and Mackaness W. (1993), Visual Access to Data Quality in Geographic Information Systems, *Cartographica*, Vol. 30, No 2/3, pp. 37–45.
- Bertin J. (1983), *Semiology of Graphics*, The University of Wisconsin Press, Madison, WI.
- Brown A. and van Elzakker C. P. J. M. (1993), The Use of Colour in the Cartographic Representation of Information Quality Generalised by a GIS, *Proceedings of 16th International Cartographic Conference*, pp. 707–720.
- Burrough P. A. (1989), Fuzzy Mathematical Methods for Soil Survey and Land Evaluation, *Journal of Soil Science*, Vol. 40, pp. 477–492.
- Burrough P. A. and Frank A. U. (1996), *Geographic Objects with Indeterminate Boundaries*, Taylor & Francis, London.
- Butenfield B. P. (1993), Representing Data Quality, *Cartographica*, Vol. 30, No. 2/3, pp. 1–7.
- Dent B. D. (1996), *Cartography, Thematic Map Design*, Wm. C. Brown Publishers.
- Fisher P. (1994), Hearing the Reliability in Classified Remotely Sensed Images, *Cartography and Geographic Information Systems*, Vol. 21, No. 1, pp. 31–36.
- Fisher P. (1996), Animation of Reliability in Computer-generated Dot Maps and Elevation Models, *Cartographic and Geographic Information Systems*, Vol. 23, No. 4, pp. 196–205.
- Kraak M. J., J. C. Muller and F. J. Ormeling (1995), GIS-Cartography: Visual Decision Support for Spatial Temporal Data Handling, *Int. J. GIS*, Vol. 9, No. 6, pp 637–645.
- Jiang B. and Kainz W. (1996), Fuzzy Overlay Analysis with Linguistic Degree Terms, in: M. J. Kraak and M. Molenaar (eds.), *Advances in GIS II*, Taylor & Francis: London, pp. 301–318.
- Jiang B., A. Brown and F. J. Ormeling (1996), Some Perceptual Aspects of Colouring Uncertainty, in: M. J. Kraak and M. Molenaar (eds.), *Advances in GIS II*, Taylor & Francis: London, pp. 477 - 490.
- McGranaghan M. (1993), A Cartographic View of Spatial Data Quality, *Cartographica*, Vol. 30, No.2/3, pp. 8–19.

- Ormeling F. J. (1995), Teaching Animated Cartography, *Proceedings of the Joint ICA Commissions Seminar on Teaching Animated Cartography*, held in Madrid 1995, Utrecht:ICA 1996, pp. 21-27.
- Schweizer D. M. and Goodchild M. F. (1992), Data Quality and Choropleth Maps: An Experiment with the Use of Colour, *Proceedings of GIS/LIS'92*, pp. 686-699.
- Smithson M. (1988), Fuzzy Set Theory and the Social Sciences: the Scope for Applications, *Fuzzy Sets and System 26*, pp. 1-21.
- Tufte E. R. (1990), *Envisioning Information*, Graphics Press, Cheshire, CN.
- Zadeh L. A. (1965), Fuzzy Sets, *Information and Control*, Vol. 8, pp. 338-353.
- Zadeh L. A. (1973), Outline of a New Approach to the Analysis of Complex Systems and Decision Process, *IEEE Trans. System, Man, and Cybernetics*, SMC-3, pp. 28-44

# CARTOGRAPHY

*The Journal of the Mapping Sciences Institute, Australia*

---

**Cartography** publishes original research papers contributing to the theory and development of the mapping sciences. The Journal also publishes reports on current applications within the mapping sciences in the "Contemporary Cartography" section.

Papers encompassing a broad range of subjects within the mapping sciences will be considered for publication *viz* cartographic design and production, computer assisted mapping, analytical and digital cartography, thematic mapping, spatial database systems, geographic and land information systems, remote sensing, cartographic history, human side of cartography, etc.

Submissions may be forwarded to:

*Cartography* Editorial Board  
PO Box 6836  
East Perth WA 6892  
Australia

Editor's email: [Wright\\_GL@cc.curtin.edu.au](mailto:Wright_GL@cc.curtin.edu.au)